Designing Low-Correlation GPS Spreading Codes via a Policy Gradient Reinforcement Learning Algorithm

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BIOGRAPHY

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ABSTRACT

With the birth of the next-generation GPS III constellation and the upcoming launch of the Navigation Technology Satellite-3 (NTS-3) testing platform to explore future technologies for GPS, we are indeed entering a new era of satellite navigation. Correspondingly, it is time to revisit the design methods of the GPS spreading code families. In this work, we develop a Gaussian policy gradient-based reinforcement learning algorithm which constructs high-quality families of spreading code sequences. We demonstrate the ability of our algorithm to achieve better mean-squared auto- and cross-correlation than well-chosen families of equal-length Gold codes and Weil codes. Furthermore, we compare our algorithm with an analogous genetic algorithm implementation assigned the same code evaluation metric. To the best of the authors’ knowledge, this is the first work to explore using a machine learning / reinforcement learning approach to design navigation spreading code signals.

I. INTRODUCTION

On January 13th of 2020, the U.S. Air Force 2nd Space Operations Squadron (2 SOPS) issued a statement that the first GPS III satellite was marked healthy and available for use [1]. This announcement officially marked the birth of the next-generation GPS constellation. In addition to broadcasting the new L1C signal, the modernized constellation is distinguished by its reprogrammable payload, which allows it to evolve with new technologies and changing mission needs.

Furthermore, with the upcoming launch of the Navigation Technology Satellite-3 (NTS-3) testing platform in 2022 [2], the United States Air Force (USAF) seeks to explore technologies which will help shape future GPS constellations [3]. NTS-3 will demonstrate the ability of the next-generation satellite-based navigation architecture and the ability to rapidly deploy new technological advancements and capabilities via the reprogrammable nature of the upcoming GPS system. Indeed, this is the third Navigation Technology Satellite (NTS) mission, with the previous two, NTS-1 and NTS-2, developed in the 1970s in order to validate technologies, including the rubidium and cesium atomic clocks [4], that were integrated into the first generation of GPS satellites launched later that decade [5]. Illustrations of the three NTS testing platforms are shown in Fig. 1.

The NTS-3 program will further test several new technologies, including new signal designs for improved GPS security and interference mitigation [3]. According to a Request for Information announcement [6], the AFRL has expressed interest in exploring modifications to all layers of the GPS signal in order to enhance PNT resiliency and performance. We are indeed entering a new era of satellite navigation. However, many of the GPS spreading codes are based on linear shift feedback registers (LFSRs) [7], which were designed decades ago before personal computers. As a result, it is time to revisit the design methods of the GPS spreading code families. In this work, we leverage recent advances in machine learning to explore a new platform for learning high-quality spreading signal families via a Policy Gradient Reinforcement Learning (RL) framework.

1. Advantages of Using Memory Codes for GPS

Currently, all broadcast GPS signals use algorithmically generable spreading code sequences. The legacy L1 C/A signal uses Gold codes, while the modernized L1C signal, to be broadcast on the GPS III satellites, uses Weil codes, along with a 7-bit pad to satisfy the desired spreading code length of 10,230 bits. Having algorithmically generable sequences was an important design consideration during the initial development phase of GPS in the 1970s due to memory storage constraints, as well as
computation limitations which restricted the ability to search for better sequences. However, today memory is an inexpensive resource, and receivers can readily store complete families of spreading code sequences. Spreading codes which are not algorithmically generable are commonly called *memory codes*, since these codes must be stored in memory.

By expanding the design space to the set of all binary sequences, we have a greater range of possible code families. Additionally, memory codes are not limited to specific lengths of code sequences. Designing a nonconforming length of spreading code signal, as was done for the GPS L2C and L5 signals, would require truncation or extension of the algorithmically generable sequences, which disturb the coveted correlation properties of these codes. Although a larger design space provides a better opportunity to find a superior set of codes, it also greatly complicates the code design method.

2. Related Prior Work

Several past works have designed memory codes using genetic algorithms (GAs) for the Galileo satellite constellation [8]. Genetic algorithms [9] are optimization algorithms which mimic the process of biological evolution. In particular, GAs maintain a population of design points, while selecting and recombining high-performing candidate solutions at each iteration.

Genetic algorithms were utilized for the Galileo E-band signals to design spreading codes which exhibit the *Autocorrelation Sidelobe Zero* property, where the auto-correlation is 0 at a relative delay of ±1 chip [8]. GAs were also utilized to design spreading code families for improved indoor positioning applications [10]. Work has also been done to define several cost parameters for selecting high quality GNSS spreading codes [11][12], including evaluating the mean squared auto- and cross-correlation components above the Welch bound [13]. Additionally, in our prior work, we proposed a multi-objective genetic algorithm platform for designing navigation spreading codes which improved on two objectives simultaneously: the mean absolute auto-correlation and cross-correlation of the code family [14].

Reinforcement learning (RL) has recently been shown to provide a promising framework for intelligent decision-making [15][16]. Indeed, reinforcement learning techniques have also been explored to design Error Correcting Codes (ECCs) [17], which are binary sequences that encode messages to improve the detection of communication transmission errors. This work explores policy gradient and advantage actor critic methods to design ECCs by optimizing the code set performance with regards to the block error rate.

3. Objective and Key Contributions

In this work, we seek to explore using reinforcement learning techniques for the application of navigation spreading code design. To the best of our knowledge, this is the first work which explores using a machine learning approach for designing navigation spreading code signals. In particular, the key contributions of our work are the following:

1. We develop a **policy gradient reinforcement learning algorithm** which constructs high-quality families of spreading code sequences.

Figure 1: Illustrations of the satellite testing platforms used in each of three Navigation Technology Satellite (NTS) initiatives. Illustration credit: Lt. Jacob Lutz, AFRL Space Vehicles Directorate.
2. We utilize a **Gaussian policy** to represent a distribution over the action space which designs the binary codes.

3. We incorporate a baseline to reduce variance in the policy gradient estimate and to improve the agent’s rate of learning.

4. We use a **maximization evaluation metric** to ensure the algorithm minimizes both the auto-correlation and cross-correlation characteristics of the spreading codes simultaneously.

With our algorithm, we demonstrate the ability to achieve low auto- and cross-correlation side peaks within the family of spreading codes. We further compare the correlation performance of the learned spreading codes with those of well-chosen families of equal-length Gold codes and Weil codes as well as with an analogous genetic algorithm implementation assigned the same code evaluation metric as our proposed algorithm.

4. **Paper organization**

The remainder of the paper is organized as follows: Section II. provides relevant technical background for the paper; Section III. describes our policy gradient reinforcement learning algorithm for the design of spreading code families; Section IV. presents our experimental validation setup and results; and Section V. concludes this paper.

II. **BACKGROUND**

1. **Binary Sequence Representation**

For mathematical convenience, we represent a binary sequence \( a^{(0,1)} \) with elements of the set \{0, 1\} as a sequence \( a \) with elements of the set \{+1, −1\} by using the following conversion

\[
\begin{align*}
\forall i : a^{(0,1)}(i) &= 0, a(i) = +1 \\
\forall i : a^{(0,1)}(i) &= 1, a(i) = -1, 
\end{align*}
\]

where \( a^{(0,1)}(i) \) and \( a(i) \) represent the \( i \text{th} \) element of the two binary sequence representations. In this paper, unless otherwise specified, we represent sequences using the \((+1, -1)\) binary representation. To denote sequences with the \((0, 1)\) binary representation, we utilize an additional superscript of \((0, 1)\), i.e. \( a^{(0,1)} \).

![Conversion Table](image)

**Figure 2:** Illustration of the conversion from the \((0, 1)\) binary sequence representation to the \((+1, -1)\) representation used for mathematical convenience. Consequently, the exclusive-OR operation between elements of the \((0, 1)\) representation becomes a multiplication operation.

The \((+1, -1)\) representation of the binary sequences simplifies the auto-correlation and cross-correlation computations performed on one or more binary sequences. In particular, an exclusive-OR operation, i.e. \( b_1 \oplus b_2 \), between two elements of the \((0, 1)\) binary representation becomes a simple multiplication operation, i.e. \( b_1 \cdot b_2 \), between two elements of the \((+1, -1)\) binary representation, as illustrated in Fig. 2.

2. **Periodic Auto- and Cross-Correlation**

Using the \((+1, -1)\) binary sequence representation defined in Eq. (1), let \( a_k \) represent the \( k \text{th} \) binary sequence in a family of length-\( \ell \) spreading code sequences. Let \( a_k(i) \in \{+1, -1\} \) represent the \( i \text{th} \) element of the sequence, where \( i \in \mathbb{I} \cap [0, \ell - 1] \) and \( \mathbb{I} \) represents the set of all integer values. Because \( a_k \) is a periodic sequence, we have that \( a_k(i) = a_k(i + \ell) \). The normalized,
periodic auto-correlation of sequence $k$ at a relative delay of $\delta$ bits is defined as

$$
\theta_k(\delta) := \frac{1}{\ell} \sum_{i=0}^{\ell-1} a_k(i) a_k(i + \delta).
$$

Similarly, in Eq. (3) we define the normalized, periodic cross-correlation between sequences $k$ and $m$ at a relative delay of $\delta$ bits as

$$
\theta_{k,m}(\delta) := \frac{1}{\ell} \sum_{i=0}^{\ell-1} a_k(i) a_m(i + \delta).
$$

3. Algorithmically Generable GPS Spreading Codes

Currently, all broadcast GPS signals use algorithmically generable spreading code sequences. The legacy L1 C/A signal uses length-1023 Gold codes, which can be generated with two 10-bit LFSRs, as shown in Fig. 3. Gold codes [18] are a type of binary spreading code sequence family which can be generated from two maximum-length sequences, or $m$-sequences. Maximum-length sequences are pseudorandom sequences of length $\ell = 2^n - 1$, which are generated using an $n$-bit linear feedback shift register (LFSR) [7].

![Diagram of the linear feedback shift register (LFSR) pair with their corresponding characteristic equations (G1 and G2).](image)

**Figure 3**: Diagram of the linear feedback shift register (LFSR) pair with their corresponding characteristic equations ($G1$ and $G2$) utilized to generate length-1023 bit Gold codes used in GPS L1 C/A. Only two 10-bit LFSRs are required for the generation of the GPS L1 C/A spreading codes.

Recently developed for the modernized GPS L1C signal, Weil codes are families of spreading codes which were initially proposed by Guohua and Quan [19] and further analyzed and proposed for the L1C signal by Rushanan [20] [21]. A Weil code family exists for any prime number sequence length $p$ and is generated from the corresponding length-$p$ Legendre sequence, constructed from the Legendre symbol [22] [23].

We construct both Gold codes and Weil codes in order to compare the performance of our proposed policy gradient-based algorithm for code generation with these spreading code families utilized in GPS. For more detailed information on how the Gold and Weil code families are defined, refer to Section VI.

4. Policy Gradient Reinforcement Learning Algorithms

In the reinforcement learning context, the goal is to teach an agent to perform actions, or follow a policy $\pi$, which maximize its total received reward. Classical reinforcement learning algorithms utilize a problem formulation which is framed as a Markov decision process (MDP) [24], defined by the tuple $(\mathcal{S}, \mathcal{A}, \mathcal{T}, \mathcal{R})$. $\mathcal{S}$ represents the state space of the environment through which the agent can traverse by executing actions $a$ from the action space $\mathcal{A}$, i.e., $a \in \mathcal{A}$. $\mathcal{T}$ represents the transition function which models state traversal in the environment. Lastly, $\mathcal{R}$ represents the reward function, which is utilized to encourage the agent to continue performing actions that are desirable for the application of interest. The reward function reflects the goal of the reinforcement learning problem and helps continuously shapes the behavior of the agent during optimization.
Policy gradient methods are a subset of reinforcement learning algorithms which directly optimize the policy function of the agent $\pi$. A policy function parameterized by $\theta$, i.e. $\pi_\theta$, represents a probability density function over the set of actions $a \in \mathcal{A}$, conditioned on the current state $s \in \mathcal{S}$ of the agent, and is more explicitly defined as
\[
\pi_\theta(a|s) := \mathbb{P}(a_t = a|s_t = s; \theta). \tag{4}
\]
This class of reinforcement learning algorithms are especially useful for problems which have large action spaces, including continuous or combinatorial action spaces, which applies to our application of interest with the design of navigation spreading codes. Policy gradient methods optimize the agent’s policy over a family of parameterized policy functions $\Pi(\Theta) := \{\pi_\theta : \theta \in \Theta\}$, by seeking to maximize the expected received reward $J$, defined as
\[
J(\theta) := \mathbb{E} \left[ \sum_{t=0} R(s_t, a_t); \pi_\theta \right] \\
= \sum_\tau \pi_\theta(\tau) R(\tau), \tag{5}
\]
where $\tau$ represents a trajectory of state-action values, $\pi_\theta(\tau)$ represents the probability of a particular trajectory given the policy parameter $\theta$, and $R(\tau)$ represents the total reward received by the agent for executing trajectory $\tau$. Thus, policy gradient methods seek to find the best policy function by solving the optimization problem
\[
\theta^* = \arg \max_\theta J(\theta). \tag{6}
\]
REINFORCE [25] is a standard policy gradient algorithm which performs Monte Carlo sampling of a set of trajectories $\{\tau^i\}$ from an initial state by executing the current policy of the agent $\pi_\theta$, also known as rollout. From the sampled trajectories, the gradient of the objective can be estimated from the log likelihood of the policy function [26] [27], defined as
\[
\nabla_\theta J(\theta) = \mathbb{E} [\nabla_\theta \log \pi_\theta(\tau) R(\tau)] \\
\approx \sum_i \nabla_\theta \log \pi_\theta(\tau^i) R(\tau^i). \tag{7}
\]
From this gradient estimate, using any first-order optimizer (e.g. stochastic gradient descent, Adam [28]), the agent improves
its policy parameter $\theta$

$$\theta \leftarrow \theta + \delta \theta$$

$$\delta \theta := g_{opt} (\nabla_\theta J(\theta)),$$

where $g_{opt}$ is the generic first-order optimizer utilized to iteratively improve the policy function of the agent. As illustrated in Fig. 4, this optimization process increases the probability of actions which lead to high rewards and decreases the probability of those which result in low rewards. Indeed, the stochasticity of the policy function leads the agent to explore more trajectories and further improve its policy.

### III. POLICY GRADIENT ALGORITHM FOR SPREADING CODE DESIGN

#### 1. Proposed Reinforcement Learning Framework

For the spreading code design application, the reinforcement learning agent is a code generator which takes in an initial set of spreading codes and follows its policy to output a modified code sequence. The resulting action is then evaluated and assigned a reward value by a code evaluator, which is utilized to update the policy parameters of the agent in order to improve its policy.

![Figure 5: Proposed reinforcement learning framework for spreading code generation. The code generator agent outputs a modified code set which is evaluated by the code evaluator for which the optimizer updates and improves the policy of the agent.](image)

Inspired by the reinforcement learning framework proposed for Error Correcting Codes by Huang et al. [17], we can model the binary spreading code design process as a discrete MDP, where the state space $S$ represents the space of spreading code families, and each state $s \in S$ is a binary sequence. The action space $A$ represents the set of bits to toggle in the current state. The transition function $T$ is a deterministic function which updates the code sequence state, and the reward function $R$ is an evaluation metric based on the auto- and cross-correlation properties of the spreading code family.

#### 2. Gaussian Policy Function Representation

Using a neural network architecture, we represent the policy function as a distribution over the set of bits to toggle from the initial code set. Thus, the policy parameters $\theta$ correspond to the hidden layers of the network. As depicted in Fig. 6, for the agent to output a set of binary codes, it must sample from this toggle distribution provided by its currently learned policy function.

![Figure 6: Illustration of policy network representation. The policy network outputs a toggle distribution, from which a sampled output code set is generated by the agent. With this policy function representation, the hidden layer values correspond to the policy parameters $\theta$.](image)
We model the set of code design policies as an uncorrelated multivariate Gaussian family, where each component corresponds to an index in the binary code sequence. Our neural network model represents this Gaussian policy function by parameterizing the mean vector of the toggle distribution via a bounded output activation function, such as the logistic or hyperbolic tangent output activations. The variance $\sigma^2$ of each component is maintained as a constant value. We can represent this probabilistic Gaussian policy function which is modeled by the network as

$$
\pi_{\theta}(a|s) = \mathcal{N} \left( \mu(\theta), \sigma^2 I^{(K\ell)} \right),
$$

where $K$ denotes the number of codes in the code family, $\ell$ denotes the length of the code family, $I^m$ represents the identity matrix of size $(m \times m)$ where $m \in \mathbb{N}$, and $\mu(\theta)$ represents the mean of the Gaussian policy function, as output by the neural network model with hidden parameters $\theta$. Note that the mean of $\pi_{\theta}$ is bounded, i.e. $\mu(\theta) \in [\mu_L, \mu_U]^K\ell$, due to the bounded nature of the output activation function of the network.

Due to the uncorrelated model of the set of Gaussian policies, each component represents a univariate Gaussian distribution over the action to be performed on each binary value in the spreading code set. In particular, the $i^{th}$ value of the action vector follows the distribution defined as

$$
\pi_{\theta}(a_i|s) = \mathcal{N} \left( \mu_i(\theta), \sigma^2 \right).
$$

The multivariate Gaussian policy function is depicted in Fig. 7. To obtain a random output code sequence from the policy network, the agent samples the action $a$ from the multi-variate Gaussian policy function defined in Eq. (9). From this sampled action $a$, the toggle transition and output code sequence is deterministically obtained by discretizing each component according to a threshold $\zeta$ defined in Eq. (12), which we choose to be the midpoint of the bounded output range of the neural network model. This deterministic transition function can be defined in an element-wise manner as

$$
s_i' = T_i(s, a) := s_i \oplus \mathbb{I} \{a_i \geq \zeta\}
$$

$$
a \sim \pi_{\theta}(a|s)
$$

$$
\zeta := \frac{\mu_L + \mu_U}{2}.
$$

Thus, if the sampled component of $a$ is above the threshold $\zeta$, the corresponding bit in the output code set $s'$ is toggled; otherwise, the corresponding bit remains the same in the output.
3. Spreading Code Evaluation Metric

In order to learn a useful set of spreading codes, the agent must learn to minimize both the auto-correlation and cross-correlation characteristics of the generated code set. The evaluation metric provides the agent feedback to improve its policy function for code generation. We utilize the mean-square non-central auto-correlation and the mean-square cross-correlation as the two objectives to minimize, defined respectively as

\begin{align}
R_{AC} &= \frac{1}{K\ell} \left( \sum_{k=1}^{K} \sum_{i=1}^{\ell-1} |\theta_k(i)|^2 \right) \\
R_{CC} &= \frac{1}{K_p\ell} \left( \sum_{k=1}^{K} \sum_{j=k+1}^{K} \sum_{i=0}^{\ell-1} |\theta_k,j(i)|^2 \right),
\end{align}

where \( K \) represents the number of sequences in the output code family \( s' \), \( \ell \) represents the length of each code sequence, \( K_p := \binom{K}{2} \) represents the number of pairs of sequences in the code family of size \( K \), \( \theta_k \) represents the auto-correlation of the \( k^{th} \) sequence in the output code set \( s' \) as defined in Eq. (2), and \( \theta_k,j \) represents the cross-correlation of sequences \( k \) and \( j \) of the output code set \( s' \) defined in Eq. (3).

In order to ensure the agent reduces both objectives, we define the reward function as the negative of the maximum of the two components from Eqs. (13) and (14)

\[ R(s') = -\max (R_{AC}, R_{CC}) \] .

Thus, in order to maximize its received reward, the agent must reduce both objectives simultaneously, without compromising one objective over the other. Note that the reward is only a function of the output sequence of the code generating agent.

4. Policy Optimization

We optimize the policy network parameters \( \theta \) by following the REINFORCE optimization process described in Section 4. To reduce the variance of the policy gradient optimization step, we incorporate a constant baseline \( b \), which is subtracted from the return [25]. Thus, with the baseline subtraction, the policy gradient estimate from Eq. (7) becomes

\[ \nabla_{\theta} J(\theta) \approx \sum_i \nabla_{\theta} \log \pi_{\theta}(\tau^i) \left( R(\tau^i) - b \right) . \]

For our constant baseline \( b \), we utilize the mean return in the current batch of samples, defined as

\[ b := \frac{1}{N} \sum_{i=1}^{N} R(\tau^i) , \]

where \( N := |\{\tau^i\}| \) represents the size of the batch of samples.

Subtracting a constant value does not introduce any bias in the gradient estimate, but it can significantly reduce its variance [25]. Indeed, by reducing the variance of the policy gradient estimate, we observe improved convergence and overall learning performance of the agent. Section IV. 2. demonstrates the learning rate of the agent with and without the incorporation of this baseline from Eq. (17).

IV. EXPERIMENTAL VALIDATION

1. Details of Experimental Setup

We validate the ability of our algorithm to devise low-correlation spreading code sequences and further compare its performance with that of well-chosen families of equal-length Gold codes and Weil codes. We compare the performance of our algorithm with Gold codes of length-63, 127, and 511. Similarly, since Weil codes only exist for sequence lengths that correspond to a prime number length, we compare our algorithm with Weil codes of length-67, 127, 257, and 521.

From our policy gradient method, we generate sequences for family sizes of \( 3 \) codes up to 31 codes. We additionally compare our proposed algorithm with the best performing code set across 10,000 sampled families of Gold codes and Weil codes. In a
few of the sample runs of Gold and Weil codes, we observed a large deviation in the auto- and cross-correlation cost components which frequently leads to worse performance on the overall objective defined in Eq. (15). In these instances, we would resample the conventional code families, leading to an improvement in the performance metric of the Gold and Weil codes.

The details of our policy gradient method are indicated in Tab. 1. We use the hyperbolic tangent function as our bounded output activation function and Adam [28] as the optimizer of the policy gradient network. We observed significant improvement in the convergence speed when fixing the input code set to a constant code sequence. Thus, without loss of generality, we fixed the input to an all-zero initial sequence.

**Table 1:** Design parameters of proposed policy gradient method.

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian policy variance ($\sigma^2$)</td>
<td>0.1</td>
</tr>
<tr>
<td>learning rate ($\eta$)</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>hidden layer size</td>
<td>$2K\ell$</td>
</tr>
<tr>
<td>number hidden layers</td>
<td>2</td>
</tr>
<tr>
<td>output activation</td>
<td>tanh</td>
</tr>
<tr>
<td>PG optimizer</td>
<td>Adam [28]</td>
</tr>
<tr>
<td>batch size ($N$)</td>
<td>100</td>
</tr>
<tr>
<td>number iterations</td>
<td>10,000</td>
</tr>
</tbody>
</table>

Furthermore, we compare the performance of our algorithm with that of an analogous genetic algorithm which is programmed to optimize over the same code maximization evaluation objective defined in Eq. (15). The genetic algorithm implementation uses fitness proportionate selection [9], which stochastically selects a candidate solution from the population with probability proportional to its normalized objective. Selected individuals are recombined using uniform crossover [29] which allows for greater recombination of candidate solutions and was observed to have better performance during initial testing than other crossover methods. We additionally incorporated elitism [30], which ensures that the genetic algorithm will continuously improve during its optimization process.

**Table 2:** Design parameters of genetic algorithm implementation.

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>elite rate</td>
<td>0.01</td>
</tr>
<tr>
<td>mutation rate</td>
<td>0.005</td>
</tr>
<tr>
<td>selection method</td>
<td>fitness proportionate [9]</td>
</tr>
<tr>
<td>crossover method</td>
<td>uniform [29]</td>
</tr>
<tr>
<td>population size</td>
<td>100</td>
</tr>
<tr>
<td>number iterations</td>
<td>10,000</td>
</tr>
</tbody>
</table>

Tab. 2 describes the parameters of the analogous genetic algorithm. For the genetic algorithm to be comparable with the policy gradient method, we set its population size to be the batch size of the policy gradient algorithm, and we set the number of iterations for both algorithms to be the same.

2. **Effect of Baseline Incorporation on Learning Performance**

Fig. 8 demonstrates the effect of incorporating the constant mean baseline on the learning performance of the agent, as measured by the normalized maximum correlation objective, i.e. $\max\{R_{AC}, R_{CC}\}$, which the agent seeks to minimize. For both of the code length scenarios shown in Fig. 8, we observe that the incorporation of the baseline by our proposed policy gradient method, shown in violet, leads to significant improvement in the learning performance of the reinforcement learning agent. This improved rate of learning with baseline incorporation is due to the reduction of the variance in the policy gradient estimate, which allows for more systematic learning and more consistent improvement on the correlation objective by the agent.
Figure 8: Training performance on the normalized maximum correlation objective, i.e. \( \max(R_{AC}, R_{CC}) \), of the policy gradient reinforcement learning agent, with (violet) and without (green) baseline incorporation. We observe that baseline incorporation leads to faster learning performance of the reinforcement learning agent.

3. Comparison with Best Performing Gold / Weil Codes

Fig. 9 shows the converged normalized mean-square auto- and cross-correlation performance of our policy gradient algorithm after training, comparing it with the best equal-length Gold and Weil code families of equal-length. We plot the final performance as a function of the code family size, with the normalized auto-correlation component \( R_{AC} \) represented by the dashed lines and the normalized cross-correlation component \( R_{CC} \) indicated by the solid lines. Because we conducted the experiments on a laptop with 16 GB of RAM, for sequences of greater than 500 bits in length, we only conducted tests for family sizes of up to 15 codes. However, by porting this algorithm on a system with more extensive computational resources, with access to GPU devices and increased RAM, we would be able to perform optimization for larger code family sizes.

Indeed, for each of the conducted tests, we observe in Fig. 9 that for all code lengths, the proposed policy gradient method in violet outperforms the best Gold code (in gold) and Weil code (in blue) families for both the auto- and cross-correlation objectives. We additionally observe in Fig. 9 that to perform well on the reward function defined in Eq. (15), the policy gradient method learns to equalize the auto- and cross-correlation objectives, in order to avoid compromising one objective for the other and perform better on the overall maximization objective.

Furthermore, Fig. 10 shows the relative performance improvement of our proposed policy gradient method with respect to the well-chosen Gold and Weil codes on maximization evaluation objective, i.e. \( \max\{R_{AC}, R_{CC}\} \). The ratio of performance improvement with respect to Gold codes is defined as

\[
\text{ratio of improvement wrt Gold codes} := \frac{f_{Gold} - f_{PG}}{f_{Gold}},
\]

where \( f_{Gold} \) represents the performance of a well-chosen Gold code family on the maximization objective and \( f_{PG} \) represents the corresponding performance of the policy gradient method for the same family size and bit sequence length. The ratio of performance improvement with respect to the Weil code results is defined similarly.

Interestingly, in Fig. 10 we generally observe an increase in the relative performance improvement as the length of the code sequence increases. This pattern could be due to the fact that the family size of the complete set of Gold codes and Weil codes correspondingly increases with the sequence length, and the policy gradient method may have greater freedom to tailor the design of the spreading code family to the desired, smaller number of sequences in the given code family. Indeed, Fig. 9 also indicates that designing spreading code sequences for larger family sizes leads to worse performance on the mean-square auto- and cross-correlation objectives for all types of code sequences.
Figure 9: Comparison of the proposed policy gradient method with that of well-chosen Gold and Weil code families as a function of family size. The normalized auto-correlation components $R_{AC}$ are represented by the dashed lines, while the normalized cross-correlation components $R_{CC}$ are indicated by the solid lines. We observe that for the conducted tests of all spreading code lengths, the policy gradient method outperforms the best Gold and Weil codes in both correlation objectives.
Figure 10: Ratio of performance improvement of policy gradient method with respect to well-chosen families of Gold and Weil codes on the maximization evaluation objective, i.e. $\max\{R_{AC}, R_{CC}\}$. We generally observe an increase in the relative improvement as the length of the code sequence increases.

4. Comparison with Analogous Genetic Algorithm + Elitism

Finally, we compare our policy gradient method with that of the analogous genetic algorithm. Fig. 11 shows separate plots of the normalized auto-correlation and cross-correlation performance for length-127 and length-257 families of spreading code sequences, while Fig. 12 shows the results for the length-511 and length-521 spreading code families. Indeed, we observe that our policy gradient method outperforms the genetic algorithm in both correlation objectives across the sequence lengths.

Figure 11: Comparison of the proposed policy gradient method with that of the analogous genetic algorithm with incorporated elitism for sequences of length-127 and 257 bits. We observe that for the conducted tests, the policy gradient method performs better than the genetic algorithm implementation in finding low-correlation spreading codes.
Figure 12: Comparison of the proposed policy gradient method with that of the analogous genetic algorithm with incorporated elitism for sequences of length-511 and 521 bits. We observe that for the conducted tests of all spreading code lengths, the policy gradient method performs better than the genetic algorithm implementation in finding low-correlation spreading codes.

V. CONCLUSION

In this work, we developed a reinforcement learning framework to devise a family of high-performing spreading code sequences which achieve low mean-square periodic auto- and cross-correlation objectives. We utilize a Gaussian policy gradient method to directly optimize the agent’s bit-toggling reinforcement learning policy, and we believe this is the first work to develop a machine learning method to design navigation spreading code signals.

We further demonstrated the ability of our algorithm to construct higher performing codes than well-chosen conventional codes, including Gold codes and Weil codes, as well as the spreading sequences obtained from a genetic algorithm with incorporated elitism. In particular, we observe that the policy gradient method outperforms these code sequences in both the auto- and cross-correlation objectives across various sequence lengths and code family sizes.
VI. APPENDIX

We construct both Gold codes and Weil codes in order to compare the performance of our proposed policy gradient-based algorithm for code generation with these spreading code families utilized in GPS. This section provides more detailed information on how the Gold and Weil code families are defined.

1. Gold Codes

With a well-chosen pair of $m$-sequences from two $n$-bit LFSRs, a corresponding family of Gold codes with length $\ell = 2^n - 1$ bits is generated, where $(n \mod 4) \neq 0$. The complete family of $K = \ell + 2$ Gold codes consists of the 2 characteristic $m$-sequences along with the set of $\ell$ codes generated by performing binary addition between the first characteristic $m$-sequence with a circularly shifted version of the second $m$-sequence, where the relative delay between the two $m$-sequences ranges from 0 to $\ell - 1$.

For a family of Gold codes, the normalized periodic auto-correlation $\theta^G_k(\delta)$ and cross-correlation $\theta^G_{k,m}(\delta)$, defined in Eqs. (2) and (3) respectively, take on one of 3 characteristic values:

$$\theta^G_k(\delta), \theta^G_{k,m}(\delta) \in \{-\frac{1}{\ell}, -\frac{\beta(n)}{\ell}, \frac{\beta(n) - 2}{\ell}\}, \forall \delta, k, m,$$

where $\beta(n) := 1 + 2\lfloor \frac{n+2}{2} \rfloor$ with $n$ representing the number of bits in the pair of LFSRs used to generate the Gold code family. We utilize $\lfloor r \rfloor$ to represent the floor function defined for any real number $r \in \mathbb{R}$ as $\lfloor r \rfloor := \sup\{r' \in \mathbb{I} : r' \leq r\}$.

2. Weil Codes

A Weil code family exists for any prime number sequence length $p$ and is generated from the corresponding length-$p$ Legendre sequence. Legendre sequences are constructed from the Legendre symbol [22] [23]. Before defining the Legendre symbol, we first must provide the definition of a quadratic residues. A value $q$ is said to be a quadratic residue (modulo $p$) if the following condition is satisfied:

$$\exists x \in \mathbb{I} \cap [1, p - 1] : x^2 \mod p = q.$$  \hspace{1cm} (20)

Otherwise, if the condition in Eq. (20) is not met, we say that $q$ is a quadratic nonresidue (modulo $p$), i.e. $q \not\equiv p$. To provide a concrete example, if we were to find the quadratic residues (modulo 7), we could evaluate the modulo of all $x \in \mathbb{I} \cap [1, p - 1]$:

$$x = 1 : 1^2 \mod 7 = 1$$
$$x = 2 : 2^2 \mod 7 = 4$$
$$x = 3 : 3^2 \mod 7 = 2$$
$$x = 4 : 4^2 \mod 7 = 2$$
$$x = 5 : 5^2 \mod 7 = 4$$
$$x = 6 : 6^2 \mod 7 = 1.$$  \hspace{1cm} (21)

Thus, from the six modulo computations in Eq. (21), we observe that the set of quadratic residues (modulo 7) is: $\{1, 2, 4\}$.

For a given prime number length $p$, the $q^{th}$ Legendre symbol is defined for values of $q \in \mathbb{I} \cap [0, p - 1]$ as

$$\left( \frac{q}{p} \right) := \begin{cases} 
0 & \text{if } q = 0 \\
1 & \text{if } q \text{ is a quadratic residue modulo } p, \text{ i.e. } q \not\equiv p \\
-1 & \text{if } q \text{ is a quadratic nonresidue modulo } p, \text{ i.e. } q \equiv p.
\end{cases}$$  \hspace{1cm} (22)

From the Legendre symbol definition in Eq. (22), the Legendre sequence is defined for indices $i \in \mathbb{I} \cap [0, p - 1]$ as

$$\text{Leg}_p(i) := \begin{cases} 
-1 & \text{if } i = 0 \\
\left( \frac{i}{p} \right) & \text{otherwise},
\end{cases}$$  \hspace{1cm} (23)

where $p$ is a prime number and corresponds to the period of the Legendre sequence. From the $(+1, -1)$ Legendre sequence definition in Eq. (23), the Weil code family of $K = 2^{\frac{p-1}{2}}$ sequences of length-$p$ is defined by performing element-wise
multiplication of the Legendre sequence with a circular shifted version. The $k^{th}$ Weil code sequence of length $p$ is defined as

$$\text{Weil}_p^k(i) := \text{Leg}_p(i)\text{Leg}_p(i + k),$$

(24)

where $k \in \mathbb{Z} \cap [1, K]$ and $K = \frac{p-1}{2}$. Note that for a Legendre with period $p$, we have that $\text{Leg}_p(i + p) = \text{Leg}_p(i)$.

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